ACKNOWLEDGEMENT

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REFLECTION OF MONODIRECTIONAL FLUX BY A COATING ON A SUBSTRATE

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NOMENCLATURE

$I(\tau, \mu)$,	intensity of radiation;
θ,	polar angle;
μ,	cosine of the polar angle θ measured from the surface normal;
τ,	optical distance;
τ_0,	optical thickness;
n_d,	refractive index of the dielectric;
ρ_D,	diffuse reflectance of the substrate;
Φ,	azimuthal angle;
Q_i,	incident flux;
q_i,	reflected attenuated flux;
$R(\mu_i', \mu_L')$,	directional reflectance due to the substrate;
$\rho_H(\mu_i')$,	directional hemispherical reflectance.

Subscripts and superscripts

c,	critical angle;
i,	incident angle;
$'$,	primes: outside the dielectric.

INTRODUCTION

MANY radiative heat-transfer problems are concerned with surfaces composed of dielectric coatings on opaque metal substrates. In working with this type of surface it is desirable to predict the reflectance of the composite structure. The intent of this paper is to present an analytical approach for prediction of the reflectance of this type of coating–substrate combination.

The model employed in this analysis assumes a monodirectional flux incident on a smooth dielectric coating, which is in turn secured to a diffusely reflecting opaque substrate. The substrate is assumed to be diffuse, since a roughened or sandblasted surface would normally be used in order to insure good bonding by the coating.

TRANSPORT EQUATION

The model employed is shown in Fig. 1. The coating and substrate are assumed to be infinite in extent with a geometrical coating thickness much greater than one wavelength. The air–coating and coating–substrate interfaces are parallel. These restrictions allow the use of axial symmetry. In addition, since the problem under consideration is concerned only with reflection, the transport equation can be reduced to Beer's law, which for the axially symmetric case can be written as

$$\mu \frac{dI(\tau, \mu)}{d\tau} = -I(\tau, \mu) \tag{1}$$

where μ is the cosine of the polar angle (θ), τ is the optical distance and $I(\tau, \mu)$ is the radiant intensity in the direction of θ and at the position τ.

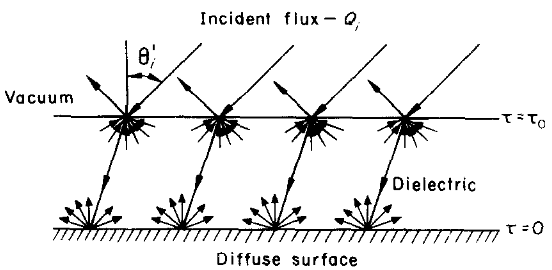

FIG. 1.

Writing the equations in terms of the intensities directed along the positive and negative directions separately, one avoids the anticipated discontinuities for $\mu = 0$ at the boundaries and equation (1) becomes

$$\mu \frac{dI^+(\tau, \mu)}{d\tau} = -I^+(\tau, \mu) \tag{2}$$

$$-\mu \frac{dI^-(\tau, \mu)}{d\tau} = -I^-(\tau, \mu) \tag{3}$$

for which the solutions are:

$$I^+(\tau, \mu) = c_1 \exp(-\tau/\mu) \tag{4}$$

$$I^-(\tau, \mu) = c_2 \exp\left(\frac{\tau - \tau_0}{\mu}\right). \tag{5}$$

The positive direction is taken in the sense of positive τ measured from the substrate.

BOUNDARY CONDITIONS

In considering a uniform flux incident upon the coating, a portion of the flux undergoes a Fresnel reflectance at the surface; the remainder is refracted into the coating. The portion which is refracted into the coating is attenuated while undergoing countless reflections between the opaque surface and the air dielectric interface. It should be noted, of ourse, that for each reflection at air–dielectric interface, a portion of the radiation escapes the dielectric.

The approach used is to account for the portion of the incident flux refracted into the dielectric separately from that reflected at the dielectric–air interface [2, 3]. The radiant flux refracted into the dielectric will be attenuated by the coating and then accounted for in the boundary conditions as it is diffusely reflected from the substrate.

Let μ' denote directions outside the dielectric and μ the directions within. The incident flux will be given by:

$$Q_i = I_i \mu_i' \Delta\mu_i' \Delta\Phi_i$$

where $\Delta\mu_i' \Delta\Phi_i$ is the solid angle containing the incident flux. The boundary conditions may be written as

$$I^-(\tau_0, \mu) = \rho(\mu) I^+(\tau_0, \mu) \tag{6}$$

$$I^+(0, \mu) = 2\rho_D \int_0^1 I^-(0, \mu)\, \mu\, d\mu + \frac{\rho_D}{\pi} \exp(-\tau_0/\mu_i) \times [1 - \rho(\mu_i')]\, I_i \mu_i' \Delta\mu_i' \Delta\Phi_i. \tag{7}$$

From Snell's Law it follows that

$$\mu = \left[1 - \frac{1}{(n_d)^2}(1 - \mu'^2)\right]^{\frac{1}{2}}. \tag{8}$$

The symbol ρ_D is the diffuse reflectance of the substrate and $\rho(\mu')$ is Fresnel's reflectance of the dielectric.

Substitution of equations (4) and (5) into equations (6) and (7) yields:

$$C_1 = 2\rho_D \int_0^1 C_2 \exp(-\tau_0/\mu)\, \mu\, d\mu + q_i \tag{9}$$

$$C_2 = \rho(\mu)\, C_1 \exp(-\tau_0/\mu) \tag{10}$$

where

$$q_i = \frac{\rho_D}{\pi} \exp(-\tau_0/\mu_i)\, [1 - \rho(\mu_i')]\, I_i \mu_i' \Delta\mu_i' \Delta\Phi_i.$$

Combining equations (9) and (10) yields:

$$C_1 = \frac{q_i}{1 - 2\rho_D \int_0^{\mu_c} \exp(-2\tau_0/\mu)\, \mu\, d\mu - 2\rho_D \int_{\mu_c}^1 \rho(\mu) \exp(-2\tau_0/\mu)\, \mu\, d\mu}. \tag{11}$$

Recalling that for $\mu \leqslant \mu_c$, $\rho(\mu) = 1{\cdot}0$, where μ_c is the cosine of the critical angle.

REFLECTANCE

The total intensity leaving the coating–air interface in the direction of μ'_L can be written as:

$$I(\mu'_L)_{\text{leaving}} = I(\tau_0, \mu'_L)_{\text{net}} + \text{Intensity of interface reflection}$$

$$I(\mu'_L)_{\text{leaving}} = \frac{1}{n_d^2}\left[I^+(\tau_0, \mu_L) - I^-(\tau_0, \mu_L)\right] + \text{Intensity of interface reflection}$$

where μ_L is the cosine of the leaving angle referred to the inside of the dielectric.

$$I(\mu'_L)_{\text{leaving}} = \frac{1}{(n_d)^2}\left[C_1 \exp(-\tau_0/\mu_L) - C_2\right] + \text{Intensity of the interface reflection.}$$

For the specular component the equation becomes:

$$[I(\mu'_L)]_{\text{spec}} = \frac{1}{(n_d)^2}\left[1 - \rho(\mu_i)\right] C_1 \exp(-\tau_0/\mu_i) + \rho(\mu'_i)\, I_i. \tag{12}$$

For all other directions the equation becomes

$$I(\mu'_L) = \frac{1}{(n_d)^2}\left[1 - \rho(\mu_L)\right] C_1 \exp(-\tau_0/\mu_L). \tag{13}$$

The intensity of the incident flux in the specular solid angle is thus the sum of a component depending on the incident flux $(I_i \mu'_i\, \Delta\mu'_i\, \Delta\Phi_i)$ which has been diffusely reflected from the substrate and the specular component whose intensity is independent of the solid angle of incidence. For purposes of representation, the authors have chosen to present the specular reflectance by a single curve for Fresnel's relationship in Fig. 2. This curve then represents the truly specular reflectance of the coating–substrate combination. Five additional curves, Figs. 3–7, represent the directional-reflectance of the combination due to the diffuse reflectance of the substrate. This reflectance is given by:

$$R(\mu'_i, \mu'_L) = \frac{\frac{1}{(n_d)^2}\left[1 - \rho(\mu_L)\right] C_1 \exp(-\tau_0/\mu_i)}{I_i \mu'_i\, \Delta\mu'_i\, \Delta\Phi_i}. \tag{14}$$

These have been plotted as $R(\mu')$ for simplification. One must then consider both the Fresnel specular component and the substrate contribution when interested in the specular direction, but need only consider the substrate contribution when interested in any direction other than specular.

The directional hemispherical reflectance would be given by:

$$\rho_H(\mu'_i) = \frac{2\pi \int_0^1 \left(\frac{1}{n_d}\right)^2 \left[1 - \rho(\mu_L)\right] C_1 \exp(-\tau_0/\mu_L)\, \mu'_L\, d\mu'_L}{I_i \mu'_i\, \Delta\mu'_i\, \Delta\Phi_i} + \rho(\mu'_i). \tag{15}$$

These values are plotted in Fig. 8.

CONCLUSIONS

As the optical thickness approaches large positive values the biangular reflectance approaches Fresnel's relation which is to be expected. It is noted from the graphs that the proposed reflectance does show the directional reflectance and the diffuse substrate contribution. This then approaches the reflectance of real coating–substrate combinations.

The coatings employed in this investigation have refractive indicies of 1·25 and 1·50. These values were chosen

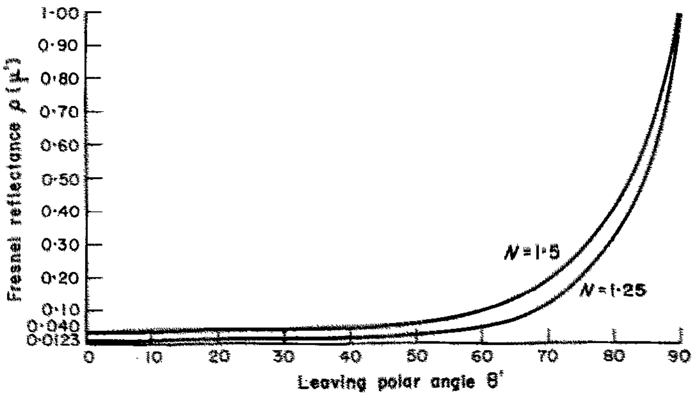

FIG. 2. Fresnel reflectance for dielectrics.

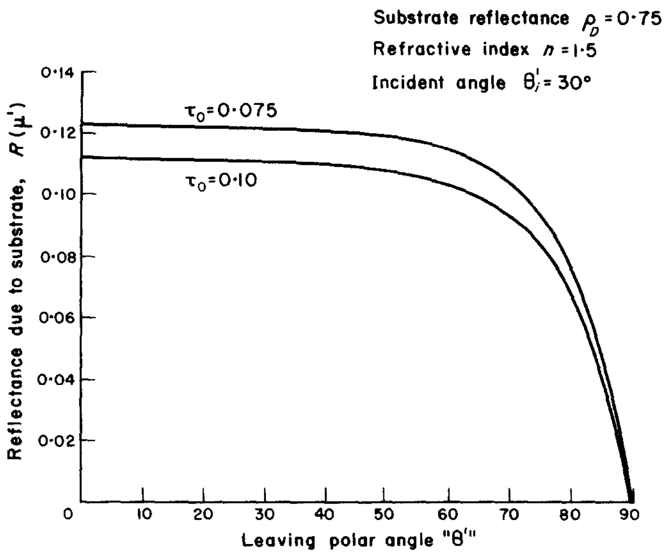
Substrate reflectance ρ_D = 0·75
Refractive index n = 1·5
Incident angle θ_i' = 30°
τ_0=0·075
τ_0=0·10
Reflectance due to substrate, $R(\mu')$
0·14
0·12
0·10
0·08
0·06
0·04
0·02
0
10
20
30
40
50
60
70
80
90
Leaving polar angle "θ'"

FIG. 3.

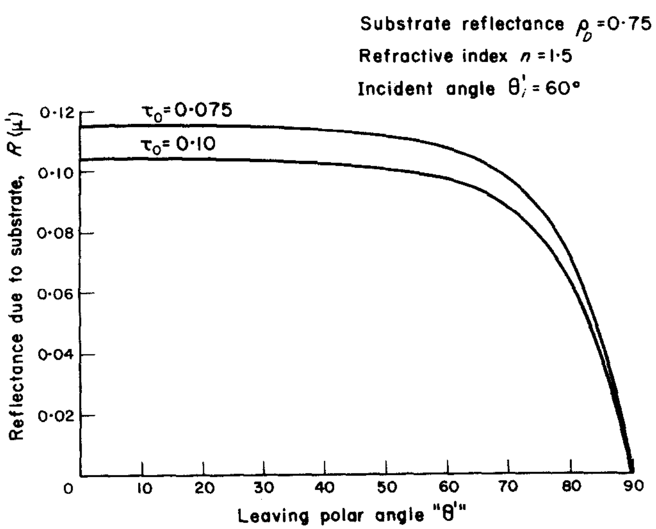
Substrate reflectance ρ_D = 0·75
Refractive index n = 1·5
Incident angle θ_i' = 60°
τ_0=0·075
τ_0= 0·10
Reflectance due to substrate, $R(\mu')$
0·12
0·10
0·08
0·06
0·04
0·02
0
10
20
30
40
50
60
70
80
90
Leaving polar angle "θ'"

FIG. 4.

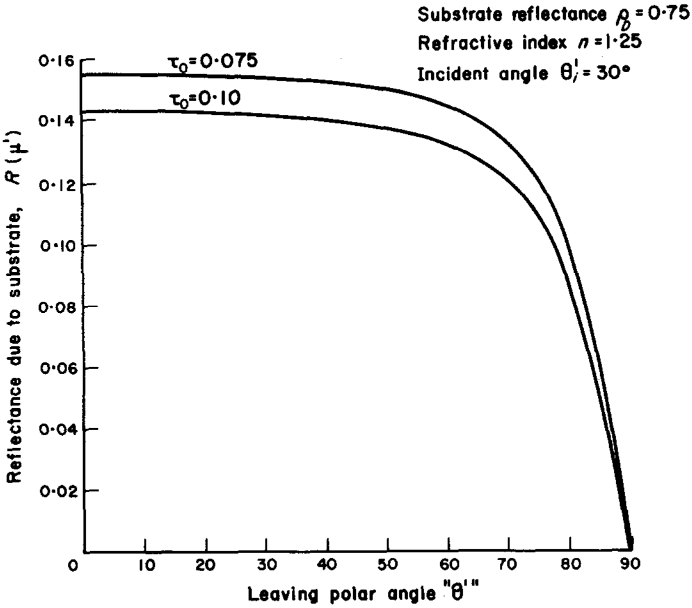

FIG. 5.

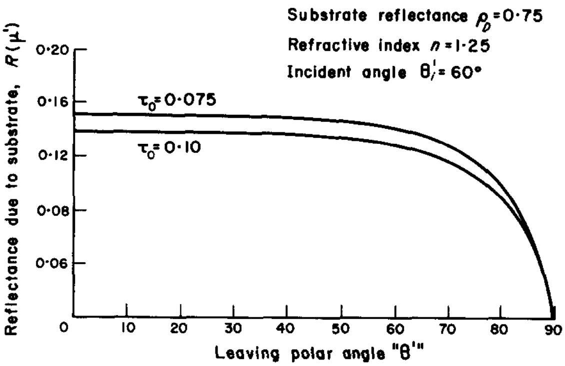

FIG. 6.

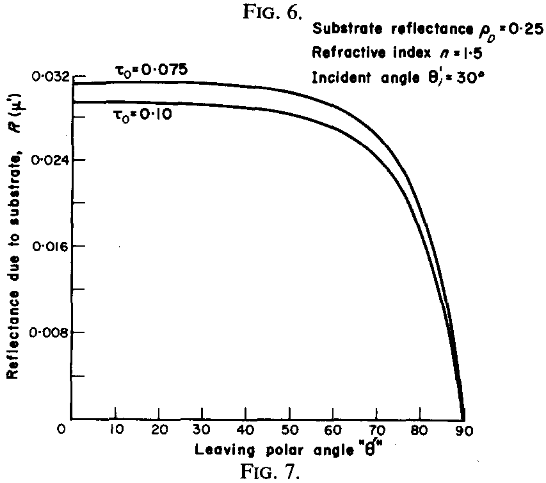

FIG. 7.

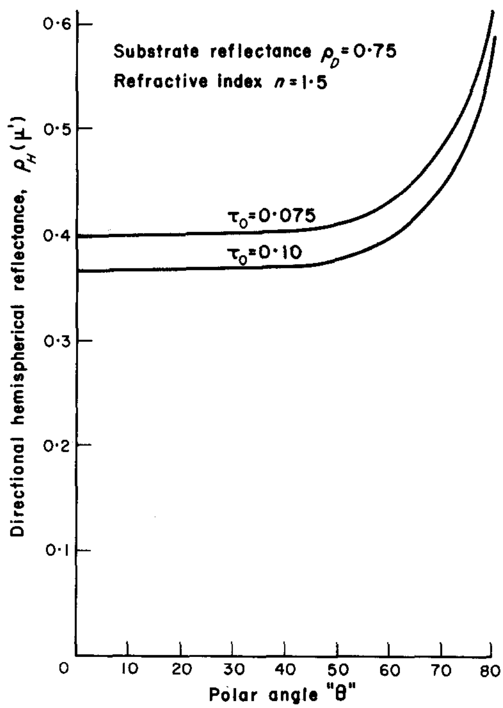

FIG. 8.

since they are somewhat indicative of the vinyl resins, nylons and cellulose derivative plastics. The diffuse conductor reflectivities of 0·75 and 0·25 cover the range from bright to heavily oxidized metals.

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TETRAFLUORETHYLENE COATINGS ON CONDENSER TUBES

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MIZUSHINA *et al.* [1] report the failure of a tetrafluorethylene coating to promote dropwise condensation of organic vapours and confirm its success with steam. This behaviour illustrates an important principle of surface chemistry which, although recognized at least qualitatively by Topper and Baer [2], is sometimes ignored in condensation studies. It is a necessary condition for dropwise condensation that the energy of the solid heat-transfer surface be less than the liquid–vapour surface tension of the condensate. The concept of a "critical surface tension, γ_c", has been used by Zisman *et al.* [3–5]. These workers have investigated the reduction in surface energy of an initially high energy surface achieved by adsorption of long chain fluorocarbon acids on to the surface as well as surface studies of polyfluorethylenes. In particular Zisman made a careful study of the contact angle, θ, of drops of liquid from a homologous series placed on the surface. He observed a linear relationship between the surface tension, γ_{LV}, of the liquid and cos θ (see Fig. 1). The critical surface tension is defined as the surface tension at which $\theta = 0$, i.e. the liquid completely wets the surface. The long chain fluorocarbon acids orientate themselves with their carboxyl group towards the metal substrate and their tails perpendicular to the surface. An acid such as n-perfluorolauric acid will display an effective surface of —CF_3 groups (and a critical wetting tension of ~6 dyn/cm at 20°C) whilst P.T.F.E. will have a structure of —CF_2–CF_2— (and a critical surface tension of 18·5 dyn/cm at 20°C). In general the degree of non-wetting is proportional to the degree of fluorination of the surface. In Table 1 the

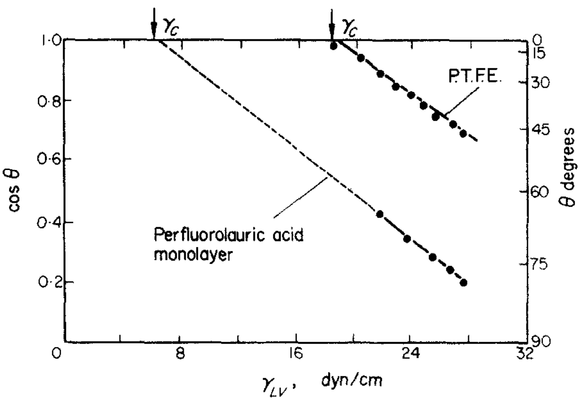

FIG. 1.

Table 1. Condensation on tetrafluorethylene-coated surfaces

Investigator	Condensate	Surface tension dyn/cm	Mode of condensation
Mizushina *et al.*	water	58·9 (at N.B.P.)	drop
	CCl_4	19·7 (at N.B.P.)	film
	CH_3OH	19·4 (at N.B.P.)	film
Topper and Baer	water	58·9	drop
	ethylene glycol	37·8	drop
	nitrobenzene	33·3	drop
	aniline	32·5	drop
	benzene	21·3	film